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Understanding entangled spins in QED

Hai-Jun Wang[1] **and W T Geng**[2]

[1] Center for Theoretical Physics, Jilin University, Changchun 130023, People's Republic of China
[2] School of Materials Science & Engineering and Department of Physics, University of Science and Technology Beijing, Beijing 100083, People's Republic of China

E-mail: whj@jlu.edu.cn

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Abstract

We have studied the stability of two entangled spins dressed by electrons by calculating the scattering phase shifts. The interaction between electrons is interpreted by fully relativistic QED and the screening effect is described phenomenologically in the Debye exponential form $e^{-\alpha r}$. Our results show that if the (Einstein–Podolsky–Rosen-) EPR-type states are kept stable under the interaction of QED, the spatial wavefunction will be constrained to be parity dependent. The spin-singlet state $s = 0$ and the polarized state $\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ along the z-axis give rise to two different kinds of phase shifts**.** Interestingly, the interaction between electrons in the spin-singlet pair is found to be attractive. Using this attraction we propose a mechanism to produce entangled pairs of massive spin-1/2 particles.

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1. Introduction

Recently, the entangled pairs dressed by massive spin-1/2 electrons in solid-state materials, which are a promising candidate for the carrier of the EPR states and hence required by quantum information processing, have been widely investigated by different groups [1–4]. Meanwhile, the quantum information theory of particle scattering becomes a newly rising topic [5]. It helps to generate the entangled pairs of electron spins. To understand the generation and evolution of the degree of entangled spins, Liu and Chen have studied the interaction between the entangled electrons [6]. By analyzing the entanglement of two identical electrons with an interaction (neither covariant nor screened) interpreted by the nonrelativistic QED, it is shown that the entangled spin-triplet states can evolve into states bearing no spin-entanglement, whereas the spin-singlet state remains *stable* in the scattering process. In a relativistic formulation, the authors in [7] have shown that the degree of entanglement is an invariant quantity. And following this approach, the authors in [8] have addressed how the

degree of entanglement transfers between spins and momenta of two particles via respectively spin-independent scattering, interaction of *lowest order* QED and passing through an exterior magnetic field. In [9], the dynamical generation of entangled spins via *nonrelativistic* and rotationally invariant interaction has been investigated in detail by analyzing the symmetric properties of scattering phase shifts.

In this paper we dedicate to the study of the *stability* of entangled spins of electrons by calculating scattering phase shifts. We will describe the intra-pair spin interaction—which is interpreted by fully *relativistic* QED—in a *complete* form covering all terms related to the spin operators, and calculate the amplitude of the scattering of two fermions belonging to an EPR pair. It is found that to make the spin-singlet state stable under the electromagnetic interaction, some limitations on the angular momentum must be added. Furthermore, the interaction between electrons in the spin-singlet pair is proved to be attractive. Such an attraction is used in a mechanism to produce entangled pairs of massive spin-1/2 particles. Results gained in this work is fundamental and will be instructive in constructing any realistic models of generating entanglement of fermions.

The remainder of the paper is arranged as follows: in the next section, we present the formulae used to calculate the phase shifts. In the third section the numerical results are presented and used to propose a mechanism to produce entangled electrons. Some concluding remarks are presented in the last section.

2. Formulism of calculating scattering phase shifts using two methods

In what follows we focus our investigation on the stability of the EPR state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ by calculating the scattering phase shifts. The method of phase shift has been widely used in many areas such as particle physics and nuclear physics, but it can only be applied to specific systems when interactions fall off more rapidly than the Coulomb potential $1/r$ [10]. In nature, as we know, it is hard to find a pure Coulomb potential. Therefore, if we are to examine the interaction between electrons that are entangled together, it is advisable to include the screening effect. Here, motivated by the idea applying Debye theory [11] to the electrolyte, fluid and dilute ions [12], we introduce a simple factor $\mathrm{e}^{-\alpha r}$ (r is the distance between the two electrons) to phenomenologically illustrate this effect. In this picture, the total potential actually falls off more rapidly than the Coulomb potential. In solid, while the electrons are moving, the crystal lattice made up of ions with positive charges and spins around the electrons will be distorted, yielding a screening effect.

To make a comparative study, two methods of directly making total spin $s = 0$ (method A) (the spin-singlet state) and polarizing the orientations of the constituent spins (method B) are employed. To avoid confusion, we use $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ to denote general $s = 0$ eigen state appeared in method A and $\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ (in fact is the combination of right-hand and left-hand *helicity* states) the polarized state along the z-axis appeared in method B. In quantum mechanics, the above two expressions usually both denote a spin-singlet state. In QED, however, since we calculate the unpolarized scattering process by averaging initial spins and summing over final spins, the results will be independent on the polarization. So, we use these two notations to compare two sorts of calculating results (due to methods A and B). Regarding entanglement, the states in methods A and B are different: the former is an entangled one but the latter is not. Because once performing a measurement (polarization) on parts (in method B), there is no possibility of making them entangled together any more. Regarding the derivation of scattering amplitude, they differ too. And these differences will make the resultant phase shifts from the two methods drastically different.

2.1. Calculation using method A

In method A, we take the spin-singlet state ($s = 0$) of two electrons as initial and final scattering states to calculate the phase shifts. To do so, we need the amplitude of the scattering process $\mathcal{M}_{fi}$, which has the following form [13]

$$\begin{aligned}\mathcal{M}_{fi} = (-\mathrm{i}e)^2 \Bigg[& \overline{u}(\mathbf{p}_3)\gamma_\mu u(\mathbf{p}_1)\overline{u}(\mathbf{p}_4)\gamma^\mu u(\mathbf{p}_2)\frac{1}{(p_1-p_3)^2} \\ & - \overline{u}(\mathbf{p}_3)\gamma_\mu u(\mathbf{p}_2)\overline{u}(\mathbf{p}_4)\gamma^\mu u(\mathbf{p}_1)\frac{1}{(p_2-p_3)^2}\Bigg] \\ = (-\mathrm{i}e)^2 & \left[\mathcal{M}_1\frac{1}{(p_1-p_3)^2} - \mathcal{M}_2\frac{1}{(p_2-p_3)^2}\right], \end{aligned} \tag{1}$$

$u(\mathbf{p})$ is the Dirac spinor defined as $\sqrt{\frac{E+m}{2E}}\left(\begin{smallmatrix}1\\ \frac{\sigma\cdot\mathbf{p}}{E+m}\end{smallmatrix}\right)$, $E = \sqrt{\mathbf{p}^2+m^2}$. Here the amplitude form is in fact an operator form, with the wavefunctions for spins determined by scattering characteristics. In this case the injected states and scattered states are not polarized, so it is necessary for the numerical evaluation to sum over the spin magnetic components m_s.

Since this amplitude is covariant, it is allowable to choose a special reference to simplify the formalism but at the same time leave the final amplitude unaltered. Here the center-of-mass(CoM) reference system is used. Then the electrons' initial momenta satisfy $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$, and the final one has $\mathbf{p}_3 = -\mathbf{p}_4 = \mathbf{q}$. For the elastic scattering process, the relation $|\mathbf{p}| = |\mathbf{q}|$ is held for the momenta. Substituting the Dirac spinors of the CoM into equation (1) and ignoring the constant $(-\mathrm{i}e)^2$ leads to the explicit forms of $\mathcal{M}_1$ and $\mathcal{M}_2$ [14]:

$$\begin{aligned}\mathcal{M}_1 = \Bigg\{ 1 + \frac{1}{(E+m)^2}[& 2\mathbf{q}\cdot\mathbf{p} + 3i(\mathbf{q}\times\mathbf{p})\cdot(\sigma_1+\sigma_2) + \mathbf{q}^2(1-\sigma_1\cdot\sigma_2) + \mathbf{q}\cdot\sigma_1\mathbf{q}\cdot\sigma_2 \\ & + 2\mathbf{q}\cdot\mathbf{p}(1+\sigma_1\cdot\sigma_2) - \mathbf{p}\cdot\sigma_1\mathbf{q}\cdot\sigma_2 - \mathbf{q}\cdot\sigma_1\mathbf{p}\cdot\sigma_2 + \mathbf{p}^2(1-\sigma_1\cdot\sigma_2) \\ & + \mathbf{p}\cdot\sigma_1\mathbf{p}\cdot\sigma_2] + \frac{1}{(E+m)^4}[\mathbf{q}\cdot\mathbf{p} + i(\mathbf{q}\times\mathbf{p})\cdot\sigma_1][\mathbf{q}\cdot\mathbf{p} + i(\mathbf{q}\times\mathbf{p})\cdot\sigma_2]\Bigg\}, \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{M}_2 = \Bigg\{ 1 + \frac{1}{(E+m)^2}[& -2\mathbf{q}\cdot\mathbf{p} - 3i(\mathbf{q}\times\mathbf{p})\cdot(\sigma_1+\sigma_2) + \mathbf{q}^2(1-\sigma_1\cdot\sigma_2) + \mathbf{q}\cdot\sigma_1\mathbf{q}\cdot\sigma_2 \\ & - 2\mathbf{q}\cdot\mathbf{p}(1+\sigma_1\cdot\sigma_2) + \mathbf{p}\cdot\sigma_1\mathbf{q}\cdot\sigma_2 + \mathbf{q}\cdot\sigma_1\mathbf{p}\cdot\sigma_2 + \mathbf{p}^2(1-\sigma_1\cdot\sigma_2) \\ & + \mathbf{p}\cdot\sigma_1\mathbf{p}\cdot\sigma_2] + \frac{1}{(E+m)^4}[\mathbf{q}\cdot\mathbf{p} + i(\mathbf{q}\times\mathbf{p})\cdot\sigma_1][\mathbf{q}\cdot\mathbf{p} + i(\mathbf{q}\times\mathbf{p})\cdot\sigma_2]\Bigg\}. \end{aligned} \tag{3}$$

Because the sign of phase shift can be determined only relatively, we determine the signs by comparing the phase shifts from the complete amplitude equation (1) and that from Coulomb interaction—for classical electrons, the S-wave Coulomb force is repulsive, and thus the corresponding phase shifts are negative. To elucidate the relationship of this complete amplitude and that due to the Coulomb interaction, we first let γ_μ in equation (1) change to γ_0, then the first part of the amplitude reduces to that of interaction for point charges (producing Mott scattering) [15]; and furthermore take the non-relativistic limit, the amplitude then can be finally described by the classical Coulomb form $\frac{1}{r}$ (producing Rutherford scattering) [15]. In the case $\gamma_\mu \to \gamma_0$ equations (2) and (3) reduce respectively to

$$\begin{aligned}\mathcal{M}_1 = \Bigg\{ 1 + \frac{1}{(E+m)^2}[& 2\mathbf{q}\cdot\mathbf{p} + \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot(\sigma_1+\sigma_2)] \\ & + \frac{1}{(E+m)^4}[\mathbf{q}\cdot\mathbf{p} + \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot\sigma_1][\mathbf{q}\cdot\mathbf{p} + \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot\sigma_2]\Bigg\}, \end{aligned} \tag{4}$$

and

$$\mathcal{M}_2 = \left\{1 + \frac{1}{(E+m)^2}[-2\mathbf{q}\cdot\mathbf{p} - \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot(\sigma_1+\sigma_2)] + \frac{1}{(E+m)^4}[\mathbf{q}\cdot\mathbf{p} + \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot\sigma_1][\mathbf{q}\cdot\mathbf{p} + \mathrm{i}(\mathbf{q}\times\mathbf{p})\cdot\sigma_2]\right\}. \tag{5}$$

The screening effect mentioned in the introduction can be phenomenologically incorporated into the propagators in equation (1) by introducing a factor $\mathrm{e}^{-\alpha r}$, where α is the inverse of the Debye screening length [11] and r the distance between the two electrons. The factor can be related to the momentum propagator by applying the Fourier transform

$$\frac{1}{(2\pi)^3}\int \frac{\mathrm{e}^{-\alpha r}}{r}\mathrm{e}^{i\vec{r}\cdot\vec{k}}\,\mathrm{d}^3\mathbf{k} = \frac{1}{k^2-\alpha^2}. \tag{6}$$

Obviously, if $\alpha \to 0$ the propagator will reduce to the original form. Now we can include the screening effect by using the propagator $1/(k^2-\alpha^2)$ instead of $1/k^2$ in equation (1). The phenomenal factor $\mathrm{e}^{-\alpha r}$ in the propagator is effective for both scalar and vector potentials, suggesting that the magnetic moments are screened in a similar way as charges. The screening effect of magnetic moments has been confirmed by Wilson *et al* in studying the Kondo effect [16]. Although the Kondo model has been studied intensively, its entanglement structure is still unclear [17]. Up to date no studies of scattering for entangled screening electrons have been reported.

As argued in the previous study [18], the Born approximation can be reasonable provided the scattering processes are of low energy and elastic ones. The scattering of electrons in solid should meet these requirements. In this approximation and in the CoM frame, the lth partial wave phase shift is expressed by the following formula [18],

$$\delta_l^J = -\tfrac{1}{2}Ek\mathcal{M}_{fi}^{Jl}(k). \tag{7}$$

Here E is the total energy of the two-electron system, $k = |\mathbf{p}| = |\mathbf{q}|$ is the magnitude of the relative momenta $\mathbf{p}$ and $\mathbf{q}$, and $\mathcal{M}_{fi}^{Jl}(k)$, with total angular momentum J and orbital angular momentum l, is the transition amplitude given by

$$\mathcal{M}_{fi}^{Jl}(k) = \frac{1}{(4\pi)^2}\sum_{m,m',m_s,m_s'} C^{JM}_{lm\frac{1}{2}m_s} C^{JM}_{lm'\frac{1}{2}m_s'} \int d\Omega(\hat{\mathbf{p}})d\Omega(\hat{\mathbf{q}})Y^*_{lm'}(\hat{\mathbf{q}})Y_{lm}(\hat{\mathbf{p}})\mathcal{M}_{fi}(\mathbf{p},\mathbf{q};m_s,m_s'), \tag{8}$$

where $C^{JM}_{lm\frac{1}{2}m_s}$ are the Clebsch–Gordan coefficients, $Y_{lm}(\hat{k})$ are the spherical harmonic functions and $\mathcal{M}_{fi}(\mathbf{p},\mathbf{q};m_s,m_s')$ the matrix elements obtained directly from equation (1) by considering the total spins in the initial and final states: $\mathcal{M}_{fi}(\mathbf{p},\mathbf{q};m_s,m_s') = \langle sm_{s'}|\mathcal{M}_{fi}|sm_s\rangle$. For special cases that $s = 0$ or $l = 0$, i.e. without spin–orbit coupling, equation (8) reduces to [19]

$$\mathcal{M}_{fi}^{l}(k) = \frac{1}{8\pi}\int_{-1}^{1}\mathrm{d}x\; P_l(x)\mathcal{M}_{fi}(\mathbf{p},\mathbf{q};x), \tag{9}$$

where $P_l(x)$ is the Legendre polynomial with $x = \cos\theta$, and θ is the angle between $\mathbf{p}$ and $\mathbf{q}$.

2.2. Calculation using method B

Now, let us evaluate the scattering phase shifts by means of method B, treating the amplitude in *helicity* representation. The calculation of polarized amplitudes is analogous to that of a previous study [13], where the spinors involved in initial and final states are specified: in

the CoM frame, the momenta of two incident particles lie along z axis, the momenta of the scattered particles lie in the y–z plane. Then for the electron–electron scattering, where the initial states are described by spinors $u(\mathbf{p}_1)$ and $u(\mathbf{p}_2)$ and scattered states by spinors $u(\mathbf{p}_3)$ and $u(\mathbf{p}_4)$, we may write [20]

$$\begin{aligned} u(\mathbf{p}_1) &= \mathrm{e}^{-\alpha_3\frac{\xi}{2}}u_1(m,0) & u(\mathbf{p}_2) &= \mathrm{e}^{\alpha_3\frac{\xi}{2}}u_2(m,0) \\ u(\mathbf{p}_3) &= \mathrm{e}^{-\mathrm{i}\sigma_1\frac{\theta}{2}}\mathrm{e}^{-\alpha_3\frac{\xi}{2}}u_3(m,0) & u(\mathbf{p}_4) &= \mathrm{e}^{-\mathrm{i}\sigma_1\frac{\theta}{2}}\mathrm{e}^{\alpha_3\frac{\xi}{2}}u_4(m,0), \end{aligned} \tag{10}$$

where the quantity ξ is defined by the hyperbolic-cosine function $ch\xi = \frac{E}{m} = \frac{1}{\sqrt{1-\mathbf{v}^2}}$, $\mathbf{v} = \frac{\mathbf{p}}{E}$, and matrices $\alpha_k = \gamma_0\gamma_k = \left(\begin{smallmatrix} 0 & \tau_k \\ \tau_k & 0 \end{smallmatrix}\right)$ in which τ_k-Pauli matrices, and $\sigma_k = \left(\begin{smallmatrix} \tau_k & 0 \\ 0 & \tau_k \end{smallmatrix}\right)$ are spin operators, $k = 1, 2, 3$. These states $u(\mathbf{p}_i)$ have been generated from the spin states in the rest frame, $(1\ 0\ 0\ 0)^T$ or $(0\ 1\ 0\ 0)^T$ (here they are written as $u_i(m,0)$, superscript T denotes a transpose operation). We have denoted the spins for $u(\mathbf{p}_1)$ and $u(\mathbf{p}_2)$ as + state if their spins are in the positive direction of z-axis, and likewise the negative direction for the $-$ state. And $u(\mathbf{p}_3)$ and $u(\mathbf{p}_4)$ being + state means that the scattered electrons have their spins along the positive direction of an axis rotated through the $\theta-$ angle from the positive direction of the z-axis. As an example, we can evaluate the matrix element $\langle + - |\mathcal{M}_{fi}| - +\rangle$ by submitting equation (10) into equation (1),

$$\begin{aligned} \langle -+|\mathcal{M}_{fi}|-+\rangle &= \overline{u}_2(m,0)\,\mathrm{e}^{\alpha_3\frac{\xi}{2}}\,\mathrm{e}^{\mathrm{i}\sigma_1\frac{\theta}{2}}\gamma_\mu^{(1)}\,\mathrm{e}^{-\alpha_3\frac{\xi}{2}}u_2(m,0) \\ &\quad\times\overline{u}_1(m,0)\,\mathrm{e}^{-\alpha_3\frac{\xi}{2}}\,\mathrm{e}^{i\sigma_1\frac{\theta}{2}}\gamma_\mu^{(2)}\,\mathrm{e}^{\alpha_3\frac{\xi}{2}}u_1(m,0)\frac{1}{2(m^2-E^2)+2p^2\cos\theta} \\ &\quad-\overline{u}_2(m,0)\,\mathrm{e}^{\alpha_3\frac{\xi}{2}}\,\mathrm{e}^{\mathrm{i}\sigma_1\frac{\theta}{2}}\gamma_\mu^{(1)}\,\mathrm{e}^{\alpha_3\frac{\xi}{2}}u_1(m,0) \\ &\quad\times\overline{u}_1(m,0)\,\mathrm{e}^{-\alpha_3\frac{\xi}{2}}\,\mathrm{e}^{\mathrm{i}\sigma_1\frac{\theta}{2}}\gamma_\mu^{(2)}\,\mathrm{e}^{-\alpha_3\frac{\xi}{2}}u_2(m,0)\frac{1}{2(m^2-E^2)-2p^2\cos\theta}, \end{aligned} \tag{11}$$

where $\gamma_\mu^{(i)}, i = 1, 2$ denote two interacting vertices, and $u_1(m,0) = (1000)^T$ and $u_2(m,0) = (0100)^T$. The subsequent steps are straightforward involving calculations of only a linear algebra, we will not explicitly show them here. Many such matrix elements such as $\langle +-|\mathcal{M}_{fi}|-+\rangle$, $\langle -+|\mathcal{M}_{fi}|-+\rangle$, $\langle +-|\mathcal{M}_{fi}|-+\rangle$ etc should be completed. The matrix elements for all possible combinations of polarized incident and scattered electrons have been listed in [13].

To compare with the results of method A, we combine the polarized states also to the entangled form $\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ and calculate the matrix element $\frac{1}{\sqrt{2}}(\langle +-| - \langle -+|)\mathcal{M}_{fi}\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle) = \langle -+|\mathcal{M}_{fi}|-+\rangle - \langle +-|\mathcal{M}_{fi}|-+\rangle$ as well as its phase shifts. Since the combination are performed after the relevant processes—such as $\langle -+|\mathcal{M}_{fi}|-+\rangle$ and $\langle +-|\mathcal{M}_{fi}|-+\rangle$—have been recognized as polarized ones, the combined stated $\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ is by no means a entangled state. For this combined polarized initial and final states, the related total angular momentum and spin angular momentum are obviously vanishing; it is assumed that equation (9) still works in this case.

3. Results and applications

The resultant phase shifts of S-, P-, D-, and f-wave from method A are plotted in figure 1. The S- and D-wave phase shifts obtained with method B are shown in figure 2.[3] The P- and F-wave shifts obtained with method B vanish for the reason we will discuss below. The phase shifts

[3] Here the natural unit 197 eV × 1 nm = 1 is used for convenience of calculation and flat-footed physical meaning. For instance, $\alpha = 1$ means the screened region of ∼200 nm.

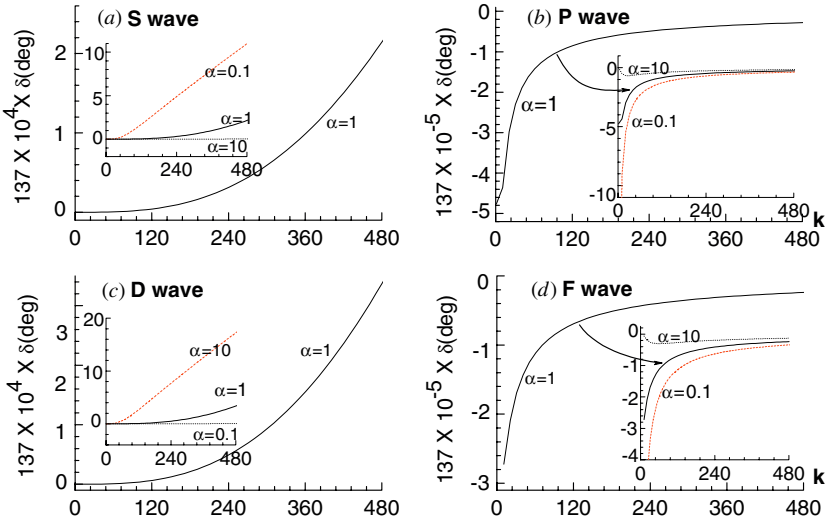

Figure 1. The calculated electron–electron scattering phase shifts for *S, P, D, F*-waves from method A. The dashed, solid, and dotted lines correspond to $\alpha = 0.1$, 1 and 10, respectively.

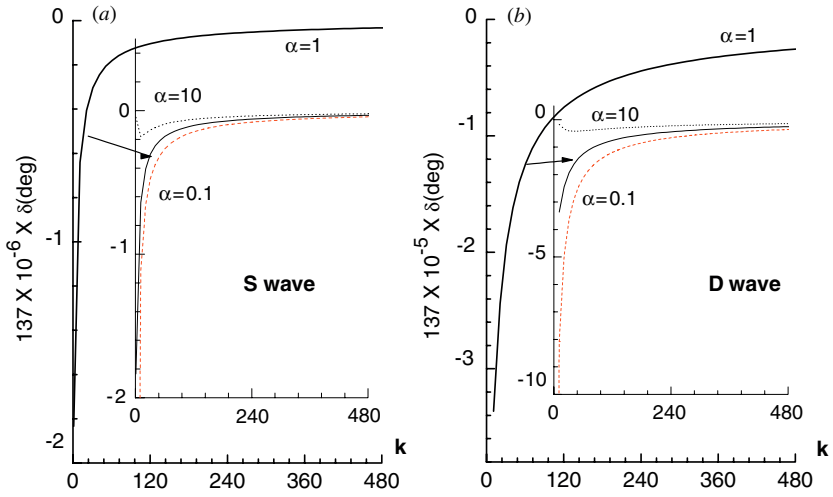

Figure 2. The calculated electron–electron scattering phase shifts for *S, D*-waves from method B. The dashed, solid, and dotted lines correspond to $\alpha = 0.1$, 1 and 10, respectively.

will not change their signs in a wide region of α provided that α is lower than the electron mass. Generally, a larger α corresponds to smaller phase shifts. The dependence is illustrated in figures 1 and 2.

Phase shifts obtained with methods A (figure 1) and B (figure 2) show much different features. The phase shifts from the two methods for any given partial wave possess different signs. The sharp difference is presumably due to the use of different 'entangled' forms in production as the scattering initial and final states. As aforementioned, the state used in method A is an entangled one but not in method B. The most salient common feature of the

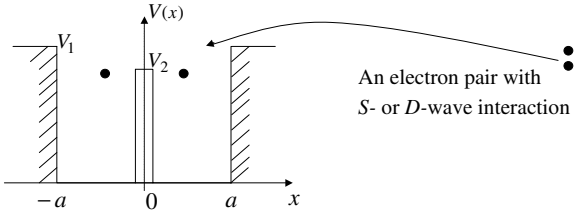

Figure 3. A square potential well with a thin square potential barrier in the middle of it, and a confined electron pair with S- or D-wave interactions. In spin singlet, the electrons attract each other and the device containing such wells will display high electrical conductivity; whereas in polarized case, the electron pairs will contribute little to conductance.

two sets of phase shifts is that they are both parity dependent. figure 1 shows that the forces of S- and D-wave are attractive and those of the P- and F-wave are strongly repulsive. We recall that according to the definition of parity, $(-1)^l$, l is the quantum number in spherical harmonics $Y_{lm}(\theta, \varphi)$, the S- and D-wave are even and the P- and F-wave are odd. In figure 2 we see that the S- and D-wave are repulsive, the P- and F-wave, however, are all vanishing. The understanding of the parity dependence may follow the fact that the even-parity spatial function of the two electrons combined with their $s = 0$ antisymmetric spin wavefunction make up of the totally perfect antisymmetric wavefunction required by identical electrons. Hence, in figure 1 the states with the P- and F-wave spacial functions are forbidden by the strong repulsive force. In figure 2, however, the forbidden states of P- and F-wave are automatically removed by the special polarization in which the spin direction and its relationship to the momenta (spatial wavefunction) are defined simultaneously. From non-relativistic QED it is impossible to obtain the parity dependence for the spatial wavefunction.

It is clear that the interaction in method A is attractive, and that in method B is repulsive. The attractive force deserves more attention. Its order of magnitudes can be evaluated directly from the resultant phase shifts. With the assumption that the phase shifts are approximately in proportion to the transmitted momentum k in the region under concern, equation (7) yields $\mathrm{d}\delta_l/\mathrm{d}k \approx -2M\langle\Psi_l|V|\Psi_l\rangle \approx -2MV/(197)^3$. For $\alpha = 1$, substituting the electron mass $M = 5 \times 10^5$ eV and $\mathrm{d}\delta_l/\mathrm{d}k \approx 10^{-8}$ gives $V \approx -10^{-8}$ eV. For a smaller α, (e.g., 0.001), in a low $\mathbf{k}$ energy region, $V \approx -10^{-4}$ eV, on just the same order of magnitudes as the force of Cooper pair in a superconductor. The results might be heuristic in the development of the spintronic devices. The contributions of each term in equations (2) and (3) to the attractive and/or repulsive forces can be numerically determined in a straightforward way. The attraction mainly comes from the contribution of $\sigma_1 \cdot \mathbf{p}_1\sigma_2 \cdot \mathbf{p}_2 + \sigma_1 \cdot \mathbf{p}_2\sigma_2 \cdot \mathbf{p}_1$, and the repulsion from $\sigma_1 \cdot \sigma_2, \sigma_1 \cdot \mathbf{p}_1\sigma_2 \cdot \mathbf{p}_1$ and $\sigma_1 \cdot \mathbf{p}_2\sigma_2 \cdot \mathbf{p}_2$. Contributions from the remaining terms such as the purely Coulomb term $\frac{1}{k^2}$ are essentially canceled out by the subtraction of two terms in equation (1).

The characteristic of phase shifts obtained with methods A and B will not change if we take two-photon processes [6] and radiation corrections into account. For the two-photon processes, a similar substraction of equation (1) holds too, so the leading contribution of the processes is canceled out. The remaining terms will not change the signs of the phase shifts because, multiplied by the square of the coupling constant, they are negligible to the next-leading-order terms of the tree level contribution. So, there is no need to consider the two-photon processes here. The radiative corrections can be done by replacing the masses and charges in scattering amplitudes with the effective ones [21]. They will not change the sign

of the calculated phase shifts for there is no electron's propagator in the amplitude. To put it another way, the screening effect can be viewed as a part of the renormalization effect [22]. Consequently, if the screening effect can be completely interpreted by renormalization, then we expect the above results would not change with a particular form of screening.

We propose a mechanism to produce entangled pairs of massive spin-1/2 particles. Electron pairs are first injected into a semiconductor [23] with the aforementioned screening environment. In a strong magnetic field spin–orbit coupling can be ignored, and then a particular relative space-wavefunction of pairs, say, S-wave or D-wave can be filtered out at the beginning by adding a strong background magnetic field. Then with an electric field voltage V the pairs can be led to a square potential well with a thin square potential barrier in the middle which divides the well into two parts (see figure 3), the well and barrier can be formed by normal semiconductor layers[4]. Only one pair is allowed in a well. The potential step of the well and the height of the dividing barrier are adjustable just as in practice. If the interaction is repulsive, the two electrons tend to be separated by the barrier and the possibility for either of them to tunnel through the barrier is low; whereas if the interaction is attractive as in the aforementioned spin-singlet state (method A), the electrons tend to stay together in one side and the barrier is easy for them to tunnel through. Thus the electrical conductivity of the well is very low in the former case and very high in the latter. In this way the pairs that make the well very conducting can be filtered out as the maximally entangled spins (in method A).

4. Conclusions

We have extensively examined the properties of the interacting entangled electrons in a fully relativistic formalism using two different calculating methods. The parity dependence of the phase shifts yielded from both approaches suggests that if the entanglement of spins is kept stable under covariant interaction, the selection of the spacial wavefunction will not be arbitrary. Furthermore, we find that the spin-singlet pair $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ and the polarized state $\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ correspond to two different types of phase shifts. In the former the electron–electron interaction is attractive and in the latter it is repulsive. In our thought, it is very important to demonstrate by deliberate calculations that there is an attraction between entangled electrons in the screening environment. The attraction of like-charged colloids confined between walls has been observed experimentally [24], and the screening electron's attraction has since been supposed to exist by some researchers [25]. However, the experimental evidence and theoretical basis for the latter were still lacking before our work report here. The attraction in principle can be tested by experiments in solid, and might be helpful in developing some devices of spintronics.

Acknowledgments

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[4] For example see figure 1 in cond-ph/9811338 and 0206323. It is a customary way to use wells or barriers in quantum mechanics to illustrated semiconductor layers made up of different materials. Of course there are many examples in textbooks or papers, we list these references at hand.

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